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Question 14-2/2: Telecommunications for e-health

STUDY GROUP 2

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TITLE: **A Brief Review on Compressive Imaging**

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ABSTRACT

Compressive Sensing (CS) is an emerging signal processing technique with various applications. Underlying the CS is uncertainty principle (UP), saying that a function or signal cannot be simultaneously sparse in time and frequency. Generalization of UP to a discrete domain implies that an *N*-length signal having *K*-degree of sparsity can be exactly reconstructed from its $M \geq CK\log(N)$ sub-samples. This paper presents a brief review of CS and its potential applications in both the spatio-temporal and transforms domain compressive imaging. Progress in this field will determine the direction of future digital electronics, including medical sensing/imaging devices.

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I. Introduction

Signal compression is an avoidable process in modern digital electronic devices, such as computers, cell-phones, and other electronics gadgets. Present day medical devices employs compression algorithm to store patient data and medical images efficiently. Two of the most prominent ones are the JPEG2000 and DICOM, in which wavelet coding is employed to compress a medical image.

The present day signal/image compression algorithm accepts digital raw data as an input and compressed signal as its output. The amount of the output data is fewer than the input; since this is the main purpose of signal compression. Accordingly, most of the collected data during the acquisition are thrown away, while a few but important ones are kept. There is an inherent inefficiency in this method since most of the collected data are discarded by the compression. It is desired that sensing process is also capable to select only important part of the data.

Signal compression is performed by first transforming the original data into a more suitable domain, in which most of the information is concentrated in a small region of support. By keeping only the data inside this region and abandoning the rest, a compression effect is obtained. An ideal sensor should have collected this small part of data. In fact, performing this task is not difficult. However, it is required that this process be adaptive since different signal responds differently under the same transform. Therefore, a sensing modality that is not depending on the input should be an ideal one.

Compressive sensing (CS) technique unifies sensing and compression in a single process. Since the observation in a CS is performed in a random fashion, the sensing process is universal. One of the most striking results of the CS is that the number of collected samples (for a given period of time) is much fewer than the one obliged by Shannon sampling theorem. Because sampling is a mandatory process in most of digital devices, which is performed by the ADC (*Analog to Digital Converter*), existence of a technique enabling faster and more efficient way will influence most of future digital devices. This paper will give a short review on the foundation/theory and application of the CS I imaging.

II. Fundamentals and Theory of Compressive Sensing

2.1 Uncertainty Principles and Signal Compression

According to Heisenberg uncertainty principle (HUP), simultaneous measurement of position and momentum cannot be made precise, that is $\Delta p\, \Delta x \geq \hbar/2$, where $\hbar = h/2\pi$ is the Planck's constant. HUP is a direct consequence of wave-particle duality. The momentum-position uncertainty can be expressed as the time-energy uncertainty $\Delta E\, \Delta t \geq \hbar/2$. According to Planck, a photon with angular frequency ω will have energy $E = \hbar\omega$. Therefore, the UP can be rewritten as

$$\Delta\omega\, \Delta t \geq 1/2 \qquad (1)$$

This equation is also called the Weyl-Heisenberg UP (WHUP). It is not only applied for physical particle, but also to any continuous function: "*a continuous-time signal cannot be simultaneously well-localized in both time and frequency*". Figure 1 shows the meaning of WHUP for a Dirac delta function.

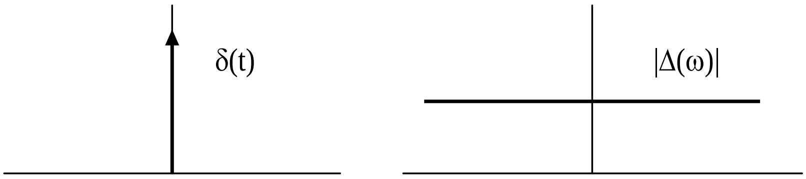


Figure 1. WHUP applied to a Dirac's delta function

Donoho and Huo [1] extend the UP to discrete domain. An N-length time-domain signal **s** with discrete Fourier transform (DFT) **S** follows the following uncertainty principle

$$|supp(s)| + |supp(S)| \geq 2\sqrt{N} \qquad (2)$$

Furthermore, Elad and Brucktein [2] obtain a more general formula for pair of basis Φ and Ψ:

$$|\Gamma_1| + |\Gamma_2| \geq 2\mu(\Phi,\Psi)^{-1} \qquad (3)$$

Where Γ_1 is the support of s in Φ-domain, Γ_2 is the support in Ψ domain, while μ(Φ,Ψ) is coherence level of both domain:

$$\mu(\Phi,\Psi) = \max_{\phi\in\Phi,\psi\in\Psi} |\langle \phi,\psi \rangle| \qquad (4)$$

An important aspect in CS is uniqueness of representation in a pair or an overcomplete basis $\{\varphi_k\}$. When the representation of a signal **s** is unique, an exhaustive search will find all of the signal ingredients α_k through L_0 minimization:

$$(P_0):\ \min \|\alpha\|_0 \text{ , s.t. } s = \sum_\gamma \alpha_\gamma \varphi_\gamma \qquad (5)$$

Furthermore, a practially computable algorithm based on L_1 require a more strict limit for uniqueness.

$$(P_1):\ \min \|\alpha\|_1 \text{ , s.t. } s = \sum_\gamma \alpha_\gamma \varphi_\gamma \qquad (6)$$

Elad, Bruckstein, and Donoho obtain the following important result for signal to be unique and practically decomposable into its ideal basis:

> **Theorem 1.** A signal s=Φα constructed from dictionary of basis Φ is unique and can be decomposed into its component through P_1 if
>
> $$\|\alpha\|_0 < 0.5(1+1/\mu) \qquad (7)$$
>
> and the solution will be identical to P_0.

Since the signal is unique, partial measurement on a proper basis is sufficient to find all of the components to reconstruct the original exactly. The noise can also be removed as well by similar method.

2.2 Compressive Sampling

Consider an *N*-length discrete time signal **s** that is expressed as an *N*-dimensional column vector. In a particular ortobasis **Ψ**, the signal can be written as $S = \Psi \cdot s$, where **Ψ** is an *N*×*N* matrix representing the basis. For most of real-world signals, one can choose a transform that has a strong decorrelation property, i.e., the ones that make most of the coefficients **S** very small, except a few numbers of them. When the magnitudes of the transform-domain coefficients are ordered, it will decay quickly. Such a signal is called *sparse* and the transform **Ψ** that enables this property is called *sparsity transform*.

In the CS, reconstruction of **s** requires just a small number of **S**. This subsampling process can be represented as projection by an *N*×*M* measurement matrix **Φ**, where *M* >>*N*. Therefore, the observable $\hat{S}$ is $\hat{S} = \Phi \cdot \Psi \cdot s = \Delta \cdot s$. The matrix Δ ≡ ΦΨ is also called the *dictionary*, which represents an over-complete basis. This equation is an *underdetermined* system of linear equations where the number of unknown is greater than the number the equations, giving non-unique solution. To solve this equation, CS assumes that the signal is sparse, which means that the number of the transform-domain coefficients, i.e. $\|\vec{S}\|_0 \equiv \sum_{n=1}^{N} |S_n|^0$ is minimum. However, this is an intractable combinatorial problem Under a particular condition given by

Theorem 1, the solution is identical to the solution of a more tractable L_1 problem by minimizing $\|\vec{S}\|_1 \equiv \sum_{n=1}^{N} |S_n|^1$ that can be recast as a convex programming problem.

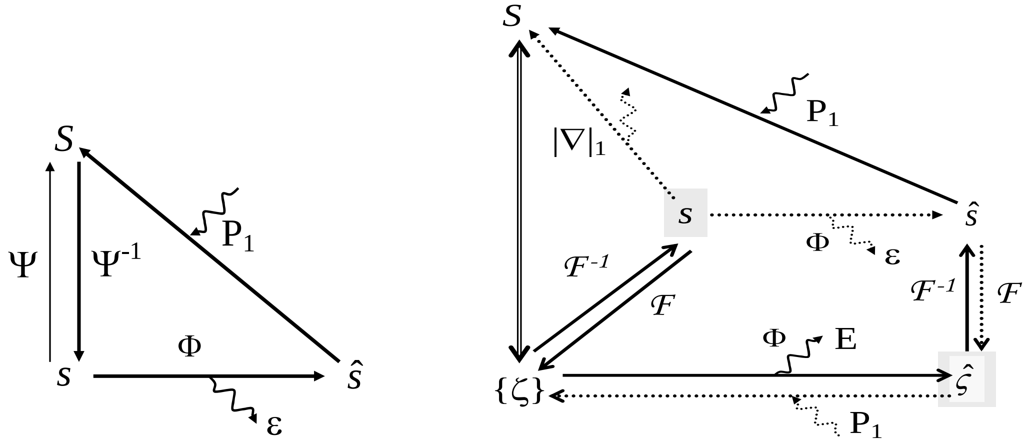
An important issue regarding this solution is that **Φ** and **Ψ** should be sufficiently incoherent. Recent findings in CS show that a general random basis has a high degree of incoherence with any basis, including the identity or spike basis **I**. Therefore, we can choose a random matrix as the projection basis Φ. In such basis, the number of required sample *M* is

$$M \geq C\cdot\mu^2(\Phi,\Psi)\cdot K\cdot log\,(N) \qquad (8)$$

where *C* is a small constant and *K* is the degree of freedom or the sparsity level of the signal or the number of non-zero coefficient of the signal when represented in the sparsity basis Ψ. For a suitable number of measured data *M* given by (8), CS guarantees to recover perfectly the time domain signal through optimization

$$\min_{\vec{S}\in R^M} \|S\|_1 \; s.t. \; \hat{s} = \langle \varphi_k, \Psi S \rangle, \; \forall m \in \{1, 2, ..., M\} \qquad (9)$$

where φ_k is a row vector of Φ. In brief, the CS principle states that for a small number of observations, it is possible to recover the original signal exactly by L_1 optimization.



(a) (b)

Figure 2. Two kinds of multi-domain diagram [3] for compressive sampling

III. Principle of Compressive Imaging and Examples

Compressive imaging employs CS to obtain two or higher dimensional picture of an object. The compressive effect is obtained when a signal **s** is projected into a measurement basis Φ. It is assumed that the signal is K-sparse in the sparsity basis Ψ. Accordingly, the observed signal $\hat{s}$ is obtained through the following operations:

$$\hat{s} = \Phi s = (\Phi\Psi)S \qquad (10)$$

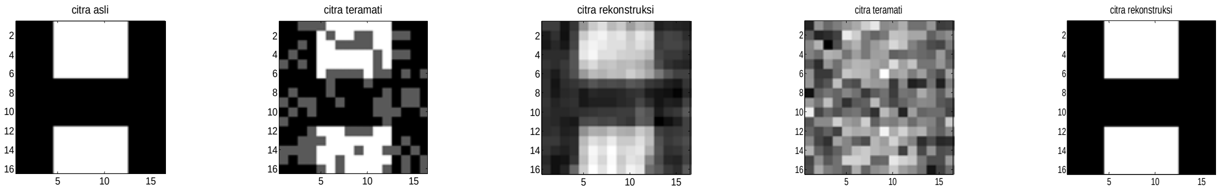
This equation can be interpreted as compression in spatio-temporal domain or reduction of basis in the (sparsity) transform domain. They have different process that can be distinguished by the multidomain diagram shown in Figure 2.

Direct interpretation of compressive sampling displayed in Fig.2(a) require (9) to seek for solution. On the other hand , Fig. 2(b) shows Fourier-domain subsampling with total variation (TV) minimization.

$$(P_1)\text{: min } |S| \text{ s.t. } \hat{s} = F^{-1}\hat{\varsigma} \qquad (11)$$

This process is rather unique since TV cannot be used in the reconstruction by inversion, instead, it is used to select a proper Fourier basis representing s with minimum TV or the smoothest one.

In the experiment, perform both of the spatio-temporal and Fourier compressive imaging are performed. In the first case, DCT basis is chosen as the sparsity transform. Original image is displayed in Figure 3(a). Compressive random sampling at a low compression level yields a distorted image shown in Fig.3.b. After performing P_1, the original image is reconstructed and shown in Fig.3(c). It is observed that the result is not satisfying, which is caused by improper choice of the sparsity transform.



(a) (b) (c) (d) (e)

Figure 3. Imaging on spatio-temporal by DCT and frequency domain by TV optimization: (a) original image, (b) observed by DCT method, and (c) reconstruction result by DCT, (d) observed TV, and (e) reconstruction by TV minimization.

The performance can be improved by choosing a proper sparsity transform, in this case the TV (Total Variance). After selection of minimum TV, corresponding Fourier coefficients is chosen to reconstruct the original image. Although the compression factor is higher than previous one, which is shown as a more distorted image in Fig.3(d), the reconstruction result is much better as displayed in Fig.3(e). In fact, an exact solution has been obtained.

IV. Conclusions

A brief review on compressive imaging has been presented. An example of spatio-temporal and Fourier-domain compressive imaging has also been demonstrated. It is found that the sparsity basis has an important role in determining reconstruction results. The similar imaging modality can be applied in medical imaging.

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